Gui-Lu Long[1,2] and Yang Sun[1,3,4]

[1]Department of Physics, Tsinghua University, Beijing 100084, P.R. China

[2]Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, P.R. China

[3]Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996

[4]Department of Physics, Xuzhou Normal University, Xuzhou, Jiangsu 221009, P.R. China

(September 27, 2000)

The projected shell model implements shell model configuration mixing in the projected basis obtained under the guidance of deformed mean field theories. Our analysis on the recently observed superdeformed band in ^{36}Ar finds that in this lightest superdeformed nucleus, the neutron and proton 2-quasiparticle and the 4-quasiparticle bands cross the ground band at same angular momentum. This constitutes a new picture of band disturbance in which the first and the second band crossing, commonly seen at separate rotation frequencies in heavy nuclei, occur simultaneously. We attempt also to understand the assumptions in the two other theoretical calculations that were previously used to interpret the same data.

PACS: 21.10.Re, 21.60.Cs, 23.20.Lv, 27.30.+t

The topic of superdeformation (SD) has been at the forefront of nuclear structure physics since observation of the first SD band in ^{152}Dy [1]. Today, superdeformation at high spin is not an isolated, but a wide-spread phenomenon across the nuclear periodic table [2], and its microscopic foundation has been established firmly. Still, it is astonishing when the recent experiment on ^{36}Ar [3] shows that, in a nuclear system with such a small number of particles (here $N = Z = 18$), it is sufficient for the quantum shell effect to stabilize the system at a superdeformation.

These new data have a large impact on theories, as they provide an ideal testing case for nuclear structure models. The ^{36}Ar SD data presented in Ref. [3] were discussed by two theoretical calculations, the Cranked Nilsson-Strutinsky (CNS) model [4] and the spherical shell model (SM) [5]. The fact that these models can give a complementary description for the SD band in ^{36}Ar indicates that they both are reasonable approaches. Nevertheless, certain assumptions were made in the two calculations. On the one hand, for a feasible SM calculation, the $1d_{5/2}$ orbital had to be excluded from the shell model space. It is known that in the quasiparticle (qp) picture appropriate for the present superdeformed minimum, the orbital $K = \frac{5}{2}$ of $1d_{5/2}$ lies very close to the Fermi levels, and it is expected that this orbital has strong correlation with other orbitals and contributes to the collective motion. It is therefore not very obvious whether the approximation of excluding $1d_{5/2}$ is a good one. On the other hand, no such exclusion is needed in the CNS calculations. However, in the CNS, pairing correlation is completely neglected although there has been no indication that pairing plays a minor role in this nucleus.

The projected shell model (PSM) [6] is a shell model truncated in the Nilsson single particle basis, with pairing correlation incorporated into the basis by a BCS calculation for the Nilsson states. More precisely, the truncation is first implemented in the multi-qp basis with respect to the deformed BCS vacuum $|0\rangle$ (see Eq. (1) below); then the violation of rotational symmetry is removed by projection [7] to form a shell model basis in the laboratory frame. Finally a shell model Hamiltonian is diagonalized in this projected space. Thus, the PSM enjoys having main advantages of mean-field theories in that it can easily build in the model the most important nuclear correlations, and at the same time, it solves the problem fully quantum mechanically and provides a good approximation to the exact shell model solution. In fact, beside systematic reproductions of energy spectra and electromagnetic transitions in normally deformed nuclei [6], it has been shown that the superdeformed bands in the $A \sim 190$ [8], $A \sim 130$ [9] and $A \sim 60$ [10] mass regions can be successfully described by the PSM.

It is clear that the PSM lies conceptually between the two approaches CNS and SM employed in Ref. [3]. In this paper, we use the PSM to analyze the new ^{36}Ar SD data. We shall show that the PSM gives comparable results of the SM in the spectrum calculation. The observed deviation from regular rotational sequence in the SD band can be understood in the PSM framework as band crossings occurring simultaneously among the ground band (g-band), 2-qp, and the 4-qp bands at same angular momentum. These 2- and 4-qp bands are based on the qausiparticles of the $1f_{7/2}$ subshell. Quantities such as B(E2), g-factor, and pairing are also studied, aiming to understand the assumptions in the two other calculations that were previously used to interpret the data [3].

In the present PSM calculation, particles in three major shells ($N = 1, 2, 3$) for both neutron and proton are activated so that the Fermi level lies approximately in the middle of the deformed single-particle states at deformation $\varepsilon_2 = 0.42$. The shell model space includes the 0-, 2- and 4-qp states:

$$|\phi\rangle_\kappa = \left\{ |0\rangle\,,\ \alpha^\dagger_{n_i}\alpha^\dagger_{n_j}\,|0\rangle\,,\ \alpha^\dagger_{p_k}\alpha^\dagger_{p_l}\,|0\rangle\,,\ \alpha^\dagger_{n_i}\alpha^\dagger_{n_j}\alpha^\dagger_{p_k}\alpha^\dagger_{p_l}\,|0\rangle \right\}, \tag{1}$$

where $\alpha^\dagger$ is the creation operator for a qp and the index n (p) denotes neutrons (protons). The projected qp-vacuum $|0\rangle$ corresponds to the g-band, whereas the projected 2- and 4-qp states to 2- and 4-qp bands, respectively. The 2- and 4-qp states are selected so that the low-lying states for each kind of configurations should be included. Were all multi-qp states considered in Eq. (1), one would obtain the full shell model space generated by particles of the three major shells.

As in the usual PSM calculations, we employ the Hamiltonian [6]

$$\hat{H} = \hat{H}_0 - \frac{1}{2}\chi \sum_\mu \hat{Q}^\dagger_\mu \hat{Q}_\mu - G_M \hat{P}^\dagger \hat{P} - G_Q \sum_\mu \hat{P}^\dagger_\mu \hat{P}_\mu, \tag{2}$$

where $\hat{H}_0$ is the spherical single-particle Hamiltonian which in particular contains a proper spin-orbit force, whose strengths (i.e. the Nilsson parameters κ and μ) are taken from Ref. [4]. The second term in the Hamiltonian is the Q-Q interaction and the last two terms the monopole and quadrupole pairing interactions, respectively. The interaction strengths are determined as follows: the Q-Q interaction strength χ is adjusted by the self-consistent relation such that the input quadrupole deformation ε_2 and the one resulting from the HFB procedure coincide with each other [6]. The monopole pairing strength G_M is taken to be $G_M = \left[19.6 - 15.7(N-Z)/A\right]/A$ for neutrons and $G_M = 19.6/A$ for protons. This choice of G_M seems to be appropriate for the single-particle space employed in the present calculation in which the major shells $N = 1, 2, 3$ are included. Finally, the quadrupole pairing strength G_Q is assumed to be proportional to G_M, the proportionality constant being fixed to 0.20 in the present work.

The eigenvalue equation of the PSM for a given spin I takes the form [6]

$$\sum_{\kappa'} \left\{ H^I_{\kappa\kappa'} - E^I N^I_{\kappa\kappa'} \right\} F^I_{\kappa'} = 0. \tag{3}$$

The expectation value of the Hamiltonian with respect to a "rotational band κ" $H^I_{\kappa\kappa}/N^I_{\kappa\kappa}$ defines a band energy. When they are plotted as functions of spin I, we call it a band diagram [6]. A band diagram displays bands of various configurations before they are mixed by the diagonalization procedure of Eq. (3). Irregularity in a spectrum may appear if a band is crossed by another one(s) at certain spin.

For the present problem, the eigenvalue equation is solved for different spins up to $I = 16$. This is the highest spin state of the SD band if the maximum spin contributed from single particles is simply counted [3]. In the context of projection, spin distribution in each basis state in Eq. (1) is given by ${}_\kappa\langle\phi|\hat{P}^I_{K_\kappa K_\kappa}|\phi\rangle_\kappa$ [11], where $\hat{P}^I_{K_\kappa K_\kappa}$ is the projection operator [7]. We have computed this quantity for each basis state and found that they approach zero for spins $I > 16$. In other words, one cannot find the spin content larger than 16 in the mean field states of the present problem. This is the band termination in the language of angular momentum projection.

Close to the neutron and proton Fermi levels of ^{36}Ar at deformation $\varepsilon_2 = 0.42$, there are four single-particle orbitals: $K = \frac{5}{2}$ of $1d_{5/2}$ and $K = \frac{1}{2}$ of $2s_{1/2}$ in the $N = 3$ shell, and $K = \frac{1}{2}$ and $\frac{3}{2}$ of $1f_{7/2}$ in the $N = 4$ shell. Thus, bands based on these orbitals are most important for determining the high spin properties of low-lying states. In Fig. 1, the band diagram is shown. Different configurations are distinguished by different types of lines, and the filled circles represent the yrast states obtained after the configuration mixing. There are about 20 bands in the calculation, but only our representative ones are displayed for discussion. Note that for the 2-qp bands, one curve represents two bands (a neutron band and a proton band) because they nearly coincide with each other for the entire spin region.

Among the 2-qp curves which start at energies of 5 – 6 MeV, one of them (dotted line) consists of two $1f_{7/2}$ quasiparticles of $K = \frac{1}{2}$ and $\frac{3}{2}$ coupled to total $K = 1$. It shows a unique behavior as a function of spin. As spin increases, it goes down first but turns up at $I = 4$. This behavior has its origin in the spin alignment of a decoupled band as intensively discussed in Ref. [6]. Because of this, it can cross the g-band at about $I = 10$. On the other hand, there is another kind of 2-qp band (long dashed curve, based on coupling of $K = \frac{5}{2}$ of $1d_{5/2}$ and $K = \frac{1}{2}$ of $2s_{1/2}$) that shows a very different behavior: it goes up nearly parallel with the g-band, and has a similar form as the g-band. This coupled band can never goes down in energy to the yrast region, thus plays a negligible role for the yrast band structure.

We have examined the other multi-qp states consisting of the $1d_{5/2}$ particles, such as the 2-qp state of $K = \frac{3}{2}$ and $\frac{5}{2}$ of $1d_{5/2}$ coupled to total $K = 1$. They lie in even higher energy regions, and have similar rotational behavior as the g-band. As far as the yrast energies are concerned, the effect of the $1d_{5/2}$ orbital on the spectrum calculations can therefore be renormalized. Contribution of the $1d_{5/2}$ orbital to the quadrupole moment can also be renormalized through effective charges. This has clarified the question why the SM reproduced the data remarkably even though it excluded the $1d_{5/2}$ orbital in the calculation [3].

The two decoupled ($K = 1$) 2-qp bands can combine to a ($K = 2$) 4-qp band which represents simultaneously broken neutron and proton pairs. In Fig. 1, this 4-qp band (solid curve) exhibits also a decoupling behavior as a function of spin, and therefore, the 4-qp band can dive into the yrast region. It is interesting to see that bands from the three different configurations (0-, 2-, and 4-qp) cross at the same place near spin $I = 10$. This is in contrast to the common band crossing picture leading to backbendings in moment of inertia. In the usual picture, one distinguishes two kinds of band crossings: the first crossing between the g-band and the 2-qp bands, and the second crossing between the 2-qp and the 4-qp bands. They cause the first and the second backbending in moment of inertia, typically seen in the rare earth nuclei at spin $I \sim 12$ and ~ 24, respectively [6].

Thus, we can interpret the band disturbance in ^{36}Ar as a consequence of the simultaneous breaking of the $1f_{7/2}$ neutron and proton pairs. After the band crossing, the yrast band gets the main component from the 4-qp band. We observe that all the (0-, 2-, and 4-qp) bands shown in Fig. 1 behave similarly at higher spins: above spin $I = 10$, all bands displayed are approximately parallel, indicating that they rotate with the same frequency.

In Fig. 2a, the PSM energy levels are compared with data, and with the SM calculations [3] in the $E(I) - E(I-2)$ plot. We observe that the PSM can reasonably reproduce the data and the results are comparable with those of the SM. Following the SD band, one sees that the discontinuity around spin $I = 10$, which corresponds to the band crossing discussed earlier, has been reproduced. Nevertheless, in contrast to the nearly perfect agreement at the low spins, the PSM calculation has small deviations from the data at the band crossing region, and for the higher spin states. For the $N \sim Z$ nuclei, there has been an open question of whether the proton-neutron pair correlation plays a role in the structure discussions. It has been shown that with the renormalized pairing interactions within the like-nucleons in an effective Hamiltoinan, one can account for the $T = 1$ part of the proton-neutron pairing [12]. However, whether the renormalization is sufficient for the complex region that exhibits the phenomenon of band crossings, in particular when both neutron and proton pair alignments occur at the same time, is an interesting question to be investigated.

Fig. 2b and 2c present the calculated B(E2) and g-factor values for the ^{36}Ar SD band. We found that the band crossing does not cause any sudden changes around the crossing spin in both quantities. In the B(E2) calculations, the effective charges are 0.5e for neutrons and 1.5e for protons, which are the same as those used in previous work and in other shell models [13]. We emphasize that employment of different effective charges can modify the absolute B(E2) values, but the essential spin dependence is determined by the wave functions. In Fig. 2b, the B(E2) values begins to decrease at spin $I = 8$, but the decrease continues in a smooth way. This is in a qualitative consistence with the CNS conclusions [3] which predicted a prolate shape with a quadrupole deformation $\varepsilon_2 \approx 0.40$ for the low spins up to $I = 8$, and then a smooth change of the values to 0.36 at $I = 16$. In the g-factor calculations, we use for g_l the free values and for g_s the free values damped by the usual 0.75 factor. The results are presented in Fig. 2c. We plot separately the neutron and proton contributions, as well as the sum of them which should be compared with measurement. We observe a smooth increase in g-factor from 0.40 at the bandhead to 0.50 at $I = 8$, and stays with this rotor value ($Z/A = 0.50$) thereafter. To test our predictions, we hope that recently developed techniques [14] can permit such g-factor measurement.

We finally show the calculated pairing gaps in Fig. 2d, in which expectation values of the pair operator are calculated by using the PSM wave functions. It is found that for this lightest SD nucleus, both neutron and proton pairing gaps are larger than 1 MeV at $I = 0$, a non-negligible value that is a comparable size of pairing gap in a heavy, deformed system. However, the pairing gaps fall quickly as rotation begins. After $I = 8$, the falling continues, but in a gentle way. The values saturate eventually with $0.3 - 0.4$ MeV. This suggests that in order to describe the low spin spectrum correctly, pairing and its dynamic evolution are important. In this particular example, it may be a reasonable approximation to neglect pairing for the high spin states. as was done in the CNS calculation [3].

In summary, the new experimental data of the SD band in ^{36}Ar, the lightest SD nucleus reported so far, has been described by the PSM. The calculated energy levels agreed well with data and with the SM results. We may thus conclude that the PSM is an efficient shell model truncation scheme, also for the well-deformed light nuclei in which the quadrupole collectivity and pairing correlations dominate. Similar conclusion has been drawn in the study of ^{48}Cr [13]. It has been found in the present case that the 0-, 2-, and 4-qp bands cross each other at about spin $I = 10$. Therefore, the 2-qp configurations do not have a chance to play a major role for evolution of the SD yrast band because the 4-qp band dominates the yrast band structure after the band crossing. Analysis of the rotational behavior for various bands in the band diagram and calculation of the pairing gaps could help us to understand the assumptions in the CNS and the SM calculations that were previously used to interpret the data. Electromagnetic properties in this SD band have been studied with predictions made for the B(E2) and g-factor values.

Communication with Dr. C. Baktash is acknowledged. Y.S. thanks the Department of Physics of Tsinghua University for the warm hospitality, and for the support from it senior visiting scholar program. This work is supported also by the Major State Basic Research Developed Program Grant No. G2000077400, the China National Natural Science Foundation Grant No. 19775026, the Fok Ying Tung Education Foundation, and the Excellent Young University Teachers' Fund of Education Ministry of China.

[1] P.J. Twin *et al.*, Phys. Rev. Lett. **57**, 811 (1986).
[2] X.-L. Han and C.-L. Wu, At. Data Nucl. Data Tables **73**, 43 (1999); B. Singh, *et al.*, Nucl. Data Sheets, **78**, 1 (1996).
[3] C.E. Svensson *et al.*, Phys. Rev. Lett. in press.
[4] T. Bengtsson and I. Ragnarsson, Nucl. Phys. **A436**, 14 (1985).
[5] E. Caurier *et al.*, Phys. Rev. **C50**, 225 (1994).
[6] K. Hara and Y. Sun, Int. J. Mod. Phys. **E4**, 637 (1995).
[7] P. Ring and P. Schuck, *The Nuclear Many Body Problem* (Springer-Verlag, New York, 1980).
[8] Y. Sun, J.-y. Zhang and M. Guidry, Phys. Rev. Lett. **78**, 2321 (1997).
[9] Y. Sun and M. Guidry, Phys. Rev. **C52**, R2844 (1995).
[10] Y. Sun, J.-y. Zhang, M. Guidry and C.-L. Wu, Phys. Rev. Lett. **83**, 686 (1999).
[11] K.H. Bhatt, S. Kahane and S. Raman, *Phys. Rev.* **C61**, 034317 (2000).
[12] S. Frauendorf and J.A. Sheikh, Nucl. Phys. **A645**, 509 (1999).
[13] K. Hara, Y. Sun and T. Mizusaki, Phys. Rev. Lett. **83**, 1922 (1999).
[14] R. Ernst *et al.*, Phys. Rev. Lett. **84**, 416 (2000).

FIG. 1. Band diagram (bands before configuration mixing) and the yrast band (the lowest band after configuration mixing, denoted by dots). Only the important lowest-lying bands in each configuration are shown.

FIG. 2. a) Transition energies $E(I) - E(I-2)$ along the SD yrast band in ^{36}Ar (The experimental data and the SM results are taken from Ref. [3]); b) calculated B(E2) values; c) calculated g-factors; d) calculated pairings, by the PSM.